Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education Examination, 2012

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3C/3D

Section Two: Calculator-assumed

Please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

Number of additional answer booklets used (if applicable):

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction tape/fluid, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Ref: 12-100

MAS3CD-S2

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of total exam
Section One: Calculator-free	7	7	50	50	33⅓
Section Two: Calculator-assumed	12	12	100	100	66⅔
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions according to the following instructions.

   Section Two: Write answers in this Question/Answer Booklet. Answer all questions.

   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked

   It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. The Formula Sheet is **not** handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed **(100 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Suggested working time: 100 minutes.

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See next page

Question 8 **(7 marks)**

The equation of a plane Π is $\sqrt{3}x + 2y - 3z = 0$ and the parametric equation of a line L is

$$L = \{x = 0, y = 1 - t, z = -t\}.$$

(a) Find the position of the point of intersection of Π and L. (1 mark)

(b) Find the size of the acute angle between L and Π. (3 marks)

(c) Find the distance of the point $Q = (0, 1, -1)$ from the plane Π. (3 marks)

See next page

Question 9 **(10 marks)**

A hot metal bar is brought to a room of constant temperature. It is known that after the metal has been in the room for t min its temperature y °C is

$$y = 23 + Ae^{-0.04t}$$

for some constant A.

(a) What is the temperature of the room? (1 mark)

(b) Show that y satisfies the differential equation (3 marks)

$$\frac{dy}{dt} + 0.04(y - 23) = 0.$$

(c) If the temperature of the bar 10 minutes after it is brought into the room is 115 °C, show that $A \approx 137$. What was the initial temperature of the bar? (2 marks)

(d) What was the instantaneous rate of change in temperature when the bar was brought to the room? (2 marks)

(e) Determine the time that elapsed for the temperature of the bar to drop to two-thirds of its initial value. Give your answer to the nearest second. (2 marks)

See next page

Question 10 **(6 marks)**

Initially the positions and velocities of a target T and two projectiles A and B are given by:

Target T: initial position $r_T = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ km; velocity $v_T = \begin{pmatrix} 7 \\ 10 \\ -3 \end{pmatrix}$ km/h

Projectile A: initial position $r_A = \begin{pmatrix} 5 \\ 28 \\ -6 \end{pmatrix}$ km; velocity $v_A = \begin{pmatrix} 6 \\ 1 \\ -2 \end{pmatrix}$ km/h

Projectile B: initial position $r_B = \begin{pmatrix} -5 \\ 20 \\ 1 \end{pmatrix}$ km; velocity $v_B = \begin{pmatrix} -3 \\ -1 \\ 7 \end{pmatrix}$ km/h.

Assuming that all the initial velocities are maintained, determine how many of the projectiles collide with the target. If collisions do occur, find the times and positions of these impacts. If collisions do not occur, state the closest distance, to the nearest kilometre, between the target and the projectile.

See next page

Question 11 **(6 marks)**

(a) Determine the equation of the tangent to the graph of $y = x^2e^x + 1, x \geq 0$ at any point where $x = a$. (2 marks)

(b) Hence or otherwise, determine the equation of the tangent T which passes through the origin. (2 marks)

(c) Evaluate the area bounded by the y-axis, the graph of $y = x^2e^x + 1$ and the tangent T to **four (4)** decimal places. (2 marks)

See next page

Question 12 **(12 marks)**

The evolution of a small population of female mammals is summarised by the following data:

Age in years	**0–1**	**1–2**	**2–3**	**3–4**	**4–5**
Birth rate	0	1.3	1.8	0.9	0.2
Survival rate	0.6	0.8	0.8	0.4	0
Initial population	194	82	55	22	6

In all parts of this question answers should be given to **three (3)** decimal places.

(a) Write down the Leslie matrix L for the population above. (1 mark)

(b) If t_n denotes the total population after n years, calculate the ratios $\frac{t_{10}}{t_9}$ and $\frac{t_{13}}{t_{12}}$. (3 marks)

(c) Using the results above, what do you notice about the population growth each year? (2 marks)

(d) After 13 years, it is decided to cull 10% of all age groups at the beginning of each year. This is equivalent to using a new Leslie matrix $L' = (1 - 0.10)L$. Find the total population at the end of the fifteenth year. (3 marks)

(e) After 15 years, the population achieved in (d) is considered ideal. The culling rate of 10% will be modified to maintain this population. What new percentage culling rate is needed? (3 marks)

See next page

Question 13 **(9 marks)**

One of the rides at a carnival involves a cabin being oscillated around a centre position with simple harmonic motion. The period of the motion is always 1.5 minutes, but the amplitude can be adjusted.

(a) There is a built-in safety feature that turns off the power when the speed reaches 80 metres/min. Determine the maximum possible amplitude of the motion. (2 marks)

(b) If the amplitude was set at 12 metres, for what percentage of the time would the speed of the cabin be greater than half of the maximum speed stated in Part (a) above? (4 marks)

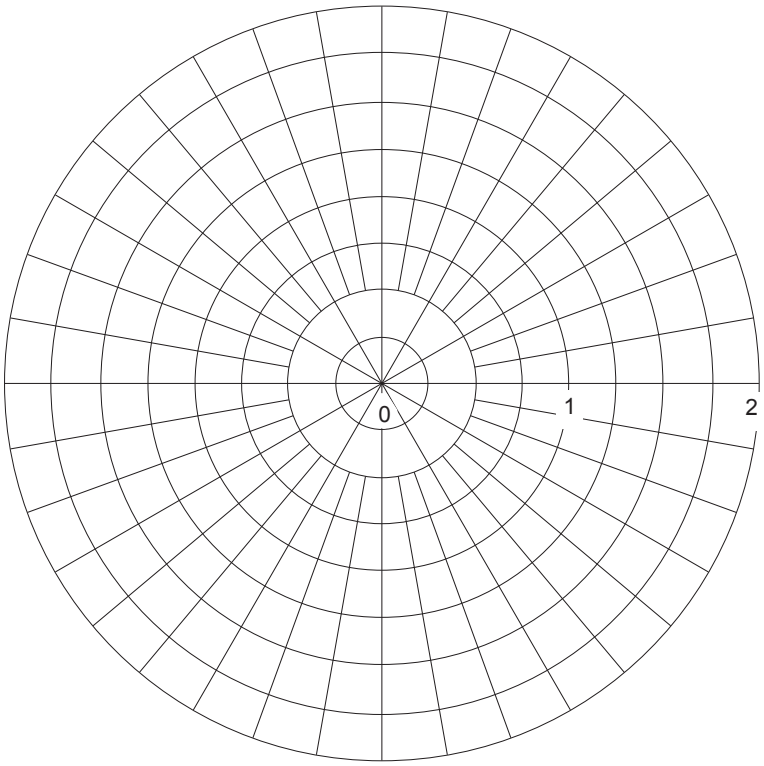
(c) On a certain day, the amplitude is set to 6 metres but the cabin is observed at $t = 0$ to be at $x = 3$ and moving away from the centre of motion. Derive an expression for the displacement of the cabin from the centre at a time t minutes later. (3 marks)

See next page

Question 14 **(8 marks)**

(a) Determine all roots of the equation $z^6 = \sqrt{3} + i$, expressing them in polar form $r\text{cis}\theta$ where $r \geq 0$ and $-\pi < \theta \leq \pi$. (5 marks)

(b) Plot all of the roots above on the diagram provided. (3 marks)



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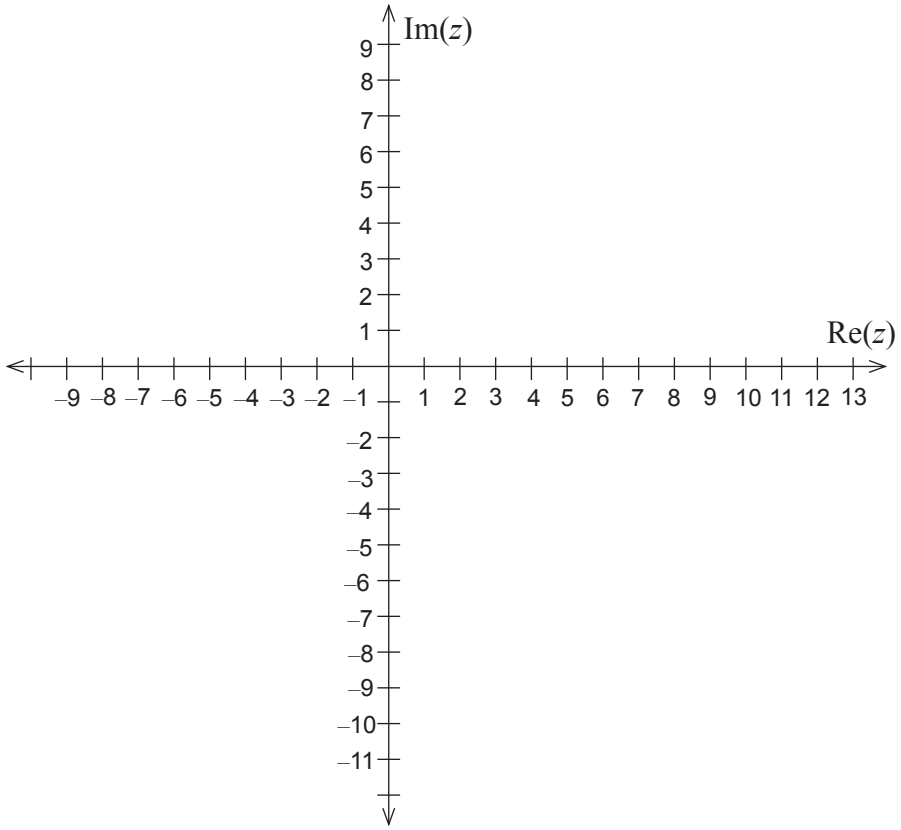
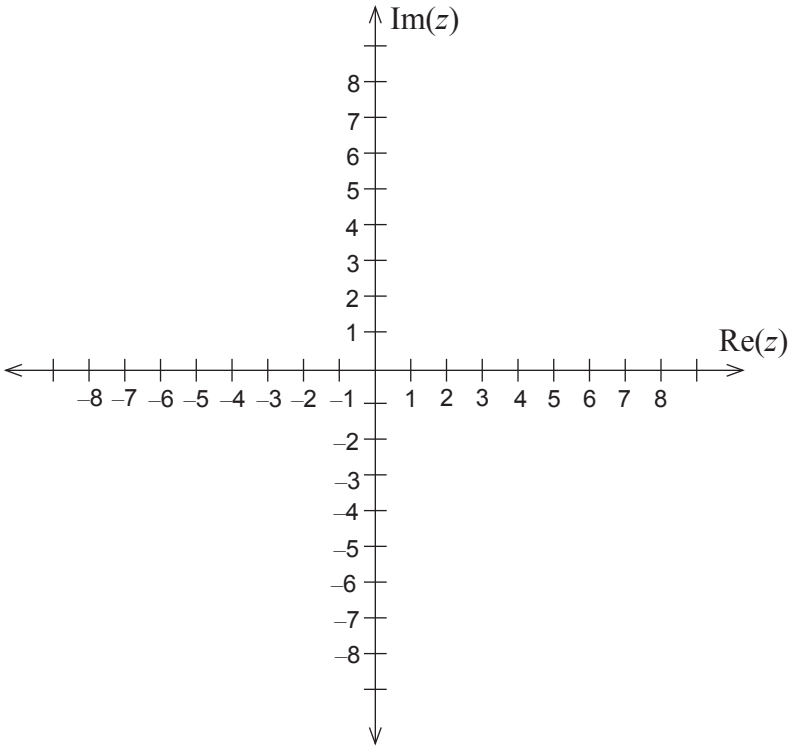
Question 15 **(10 marks)**

(a) Sketch the following regions in the complex plane.

(i) $|z - 3 + 4i| \leq 7$ (3 marks)



(ii) $\text{Im}\, z \geq 2\, \text{Re}\, z - 1$ (3 marks)



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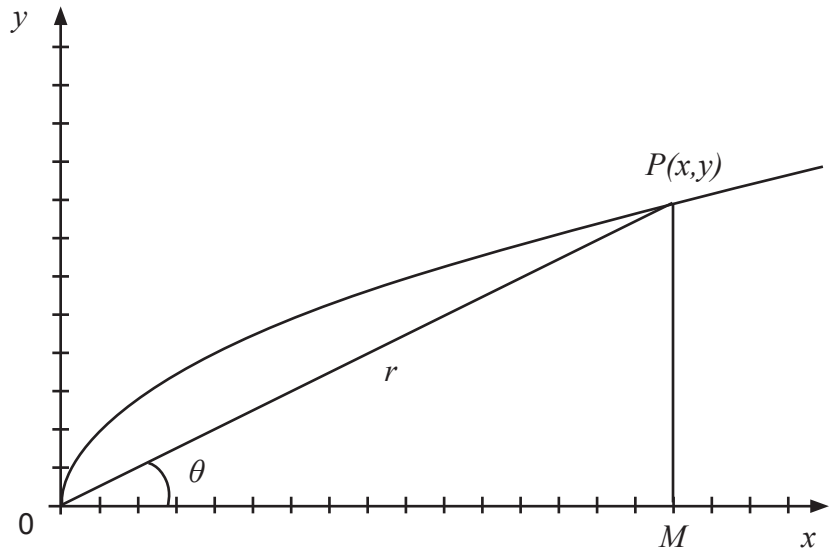
(b) The set of points in the complex plane that satisfy $|z - 3 - i| = |z - a - bi|$, where a and b are certain real constants, can alternatively be defined by the property that they lie on the line $\text{Im}\, z = -2\, \text{Re}\, z + 17$.

Determine the values of a and b. (4 marks)

See next page

Question 16 **(8 marks)**

A particle moves along the parabolic path $y^2 = x$ in a way such that the y-coordinate of its position increases at a constant rate of 10 m/s. It starts at the origin O and t seconds later is located at the point $P(x,y)$.



When at P, the particle is at a distance r metres from O and the line OP is inclined at an angle θ radians to the horizontal.

(a) Show that $\tan\theta = \frac{1}{y}$ and $r = y\sqrt{1+y^2}$. (3 marks)

See next page

(b) When the particle is 2 metres above the horizontal line OM

(i) at what rate is θ changing? (3 marks)

(ii) at what rate is its distance from O changing? (2 marks)

See next page

Question 17 **(5 marks)**

Let $y = a^x$ where a is a positive constant.

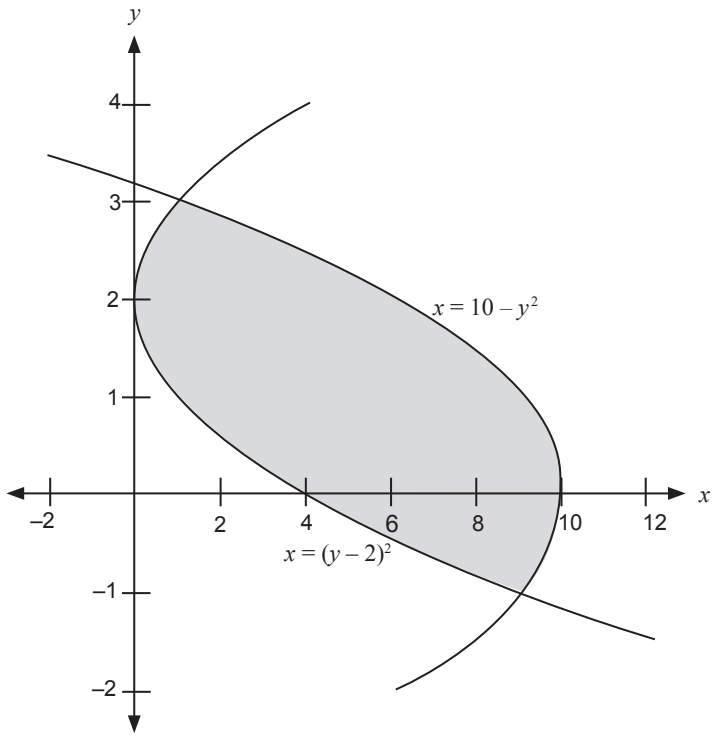
(a) Determine $\frac{dy}{dx}$. (2 marks)

(b) Prove that $\int_0^1 y\,dx = \frac{a-1}{\ln a}$. (3 marks)

See next page

Question 18 **(8 marks)**

(a) Determine the shaded area between the curves below. (4 marks)



(b) Determine the values of the real constants p,q and r if the equation $|3x + 6| = p|x + q| + r$ is satisfied for all $x \in [-2, 3]$ but for no other real values. (4 marks)

See next page

Question 19 **(11 marks)**

(a) Prove that $\text{cis}\,\theta\,\text{cis}\,\varphi = \text{cis}(\theta + \varphi)$. (You should not assume that $\text{cis}\,\theta = e^{i\theta}$.) (3 marks)

(b) By considering $(5 + i)^4 (1 - i)$, or otherwise, prove that

$$4\tan^{-1}\frac{1}{5} = \frac{\pi}{4} + \tan^{-1}\frac{1}{239}.$$ (4 marks)

See next page

(c) Let $z_1 = \text{cis}\ \theta$ and $z_2 = \text{cis}\ \varphi$, such that $|z_1 - z_2| = |z_1 + z_2|$. (4 marks)

(i) Show that $Re(z_1 \overline{z_2}) = 0$.

(ii) Hence, or otherwise, deduce that $|\theta - \varphi| = \frac{\pi}{2}$.

End of questions

Additional working space

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